Quantum State and Process Tomography with Entangled Photons

David Branning, *Univ. Illinois, Urbana-Champaign*

Paul Kwiat, J. Altepeter, E. Jeffrey, N. Peters, A. VanDevender

P. Goldbart, T.C. Wei

D. James (LANL), A. White (Univ. Queensland)

Optical Approaches to Quantum Information

The polarization of a photon comprises an ideal 2-state quantum system, equivalent to any other 2-state quantum system (e.g., two-level atom, spin-1/2, etc.), but often much easier to produce, manipulate, and detect.

- Quantum communication – q. cryptography, dense coding, teleportation
- Quantum "network" – coupling together remote quantum computers, multiparty protocols
- Small-scale quantum circuits – algorithms, repeaters, DFS studies, simulating other systems
- *Scalable* quantum computing?

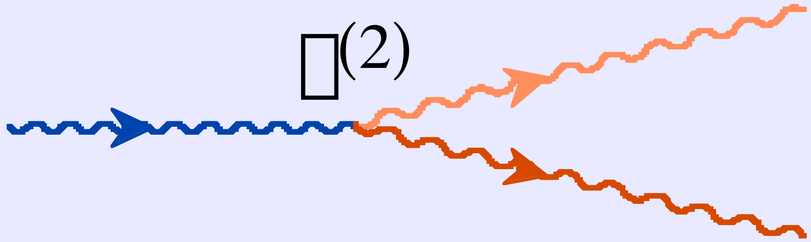
Current goals:
- push the limits on control and measurement (error $< 10^{-4}$)
- develop a *"benchmark technology"*
- explore fundamentals of quantum info. processing.

Basic tools: arbitrary quantum state synthesis, precise process control, accurate q. tomography

Advanced tools: quantum storage, single-photon sources, entanglement-on-demand, etc.

Spontaneous Parametric Downconversion

Burnham & Weinberg, PRL **25**, 84 (1970)



$\omega_p = \omega_s + \omega_i$ *

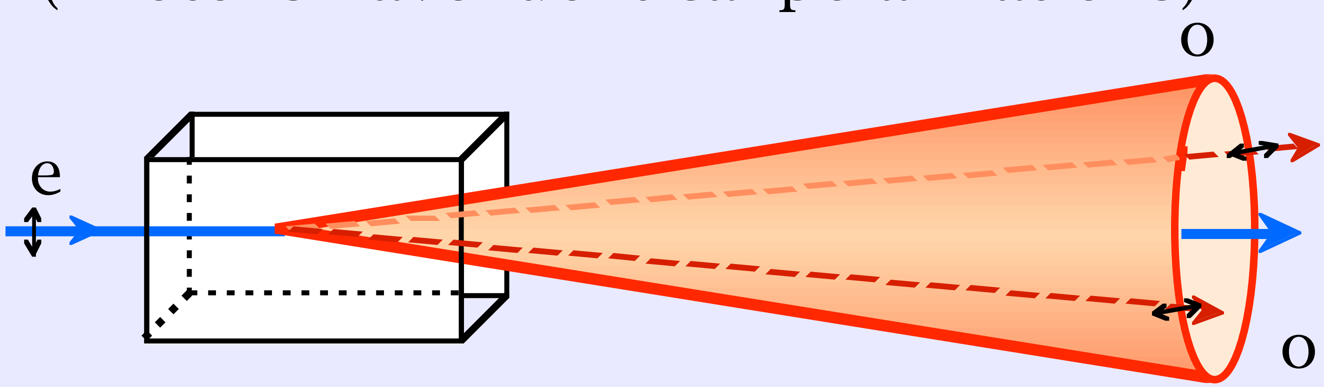
$\vec{k}_p = \vec{k}_s + \vec{k}_i$ †

*Energy conservation → energy entanglement

†Momentum conservation → momentum entanglement

Type-I Phase-matching

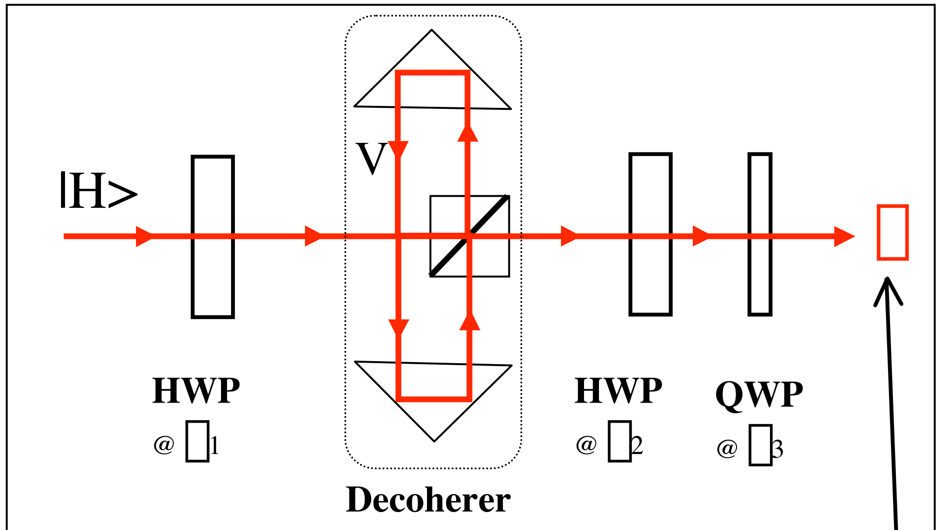
(Photons have identical polarizations)

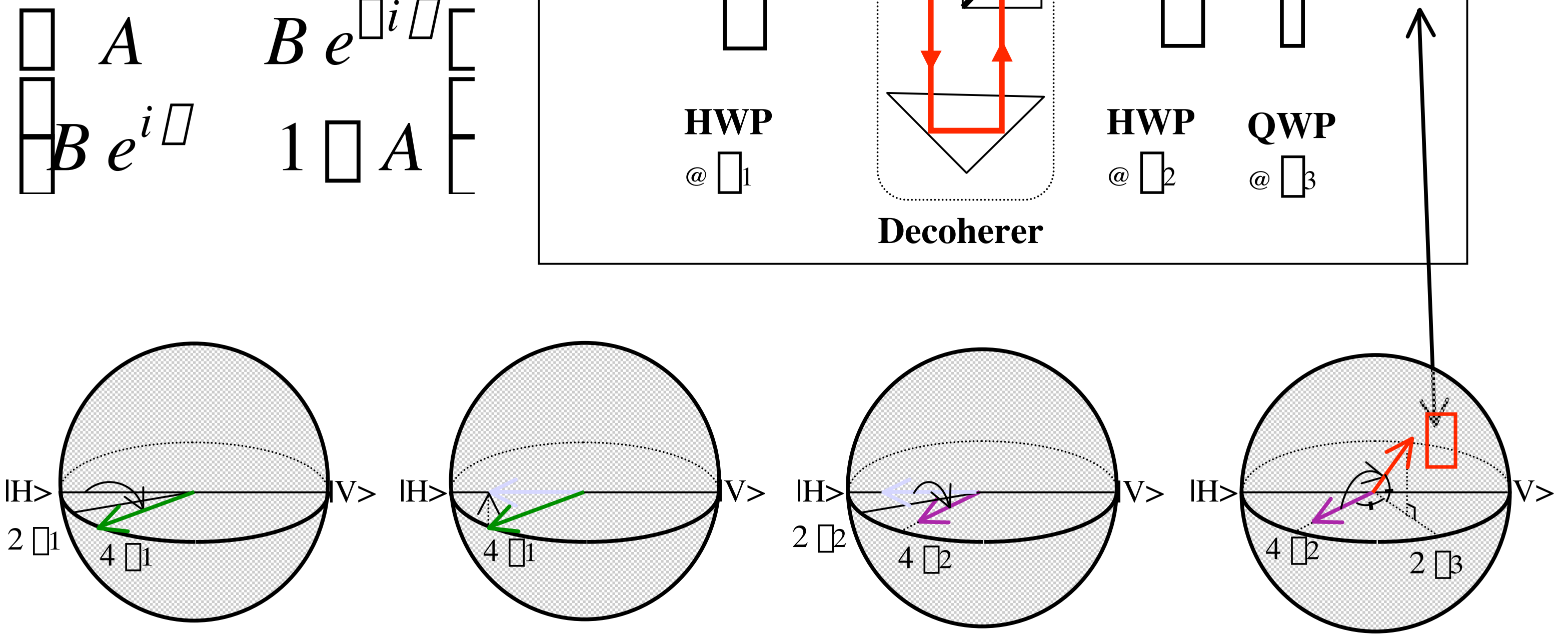


Single Qubit Generation

An arbitrary state:

$$\rho = \begin{pmatrix} A & B\,e^{-i\phi} \\ B\,e^{i\phi} & 1-A \end{pmatrix}$$





$$\theta_1 = \frac{1}{4} ArcCos\left[\sqrt{(2A-1)^2 + 4B^2}\right]$$

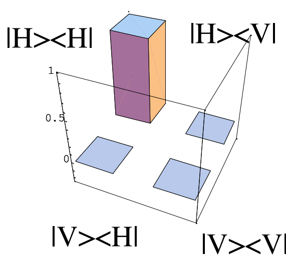
$$\theta_2 = \frac{1}{4}\left(ArcTan\left[\frac{2B\,Cos[\phi]}{2A-1}\right] + ArcTan\left[\frac{2B\,Sin[\phi]}{\sqrt{(2A-1)^2 + 4B^2\,Cos^2(\phi)}}\right]\right)$$

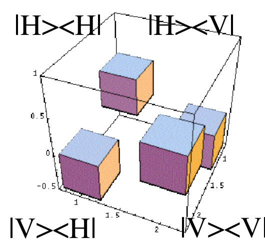
$$\theta_3 = \frac{1}{2} ArcTan\left[\frac{2B\,Cos[\phi]}{2A-1}\right]$$

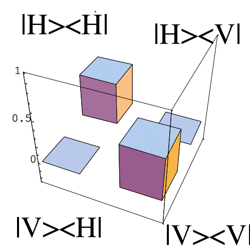
Experiment vs. Theory

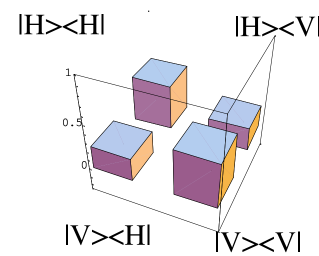
State	$\vert H\rangle$	$\vert \bar{D}\rangle$	Mixed	Partially Mixed
ρ	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0.5 & 0.25 \\ 0.25 & 0.5 \end{pmatrix}$

Theory ρ

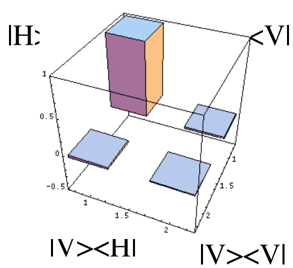


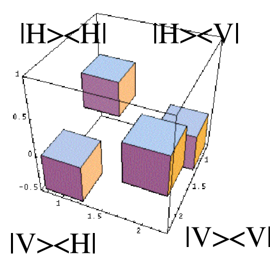


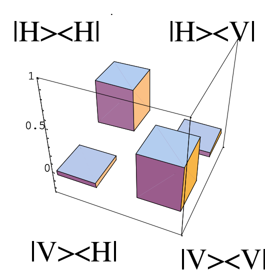


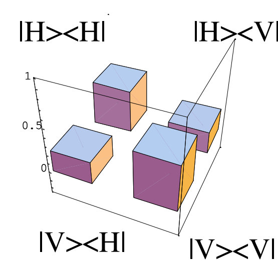


Experiment ρ



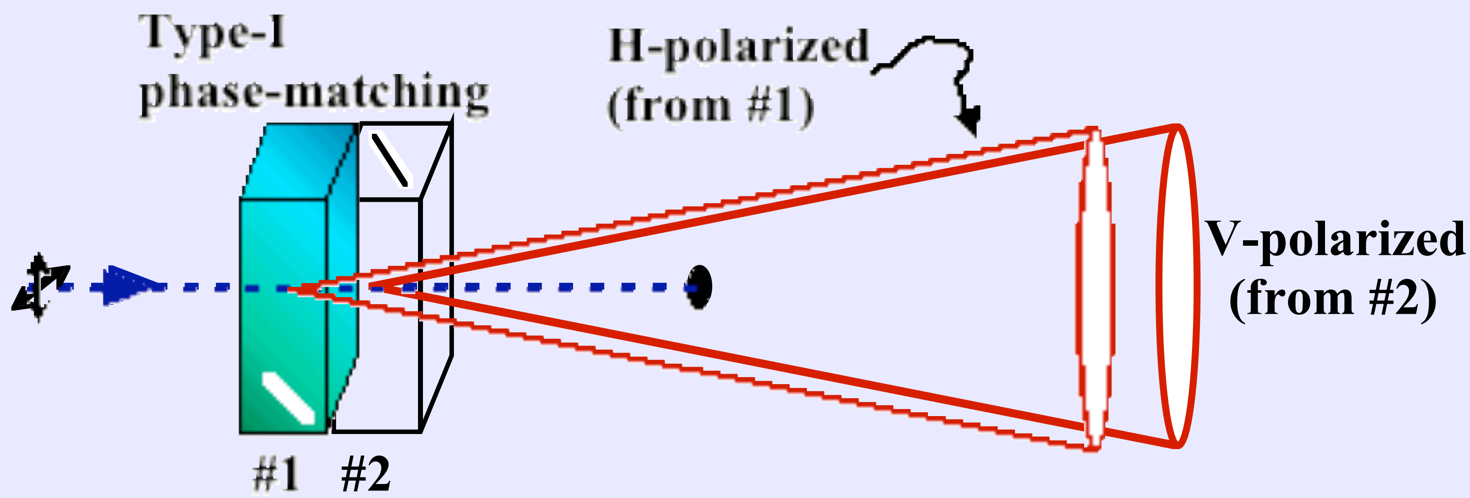






Fidelity	**0.998**	**0.996**	**0.998**	**0.999**

Two-crystal Source



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1 |H\rangle_2 + e^{i\varphi}|V\rangle_1 |V\rangle_2\right)$$

Maximally-entangled state

Arbitrary two-qubit pure states

- Via spontanous parametric downconversion (PDC), our experimental system can generate pure states of the form $\cos\theta|\mathrm{HH}\rangle + \sin\theta e^{i\phi}|\mathrm{VV}\rangle$ where the two angles are tunable by waveplate settings before PDC
- We can obtain arbitrary states of the form a|HH>+b|HV>+c|VH>+d|VV>, by preparing a state with appropriate θ and ϕ followed by a series of waveplates in both arms (i.e., local unitary transformations)
- Mathematically, given (a,b,c,d) (properly normalized), we can find (θ, ϕ) and local unitaries (U^A, U^B) such that $U_1 \otimes U_2(\lambda,0,0,\mu)^T = (a,b,c,d)^T$
- Solution:

$$|\psi\rangle = a|\mathrm{HH}\rangle + b|\mathrm{HV}\rangle + c|\mathrm{VH}\rangle + d|\mathrm{VV}\rangle = \sqrt{p}\,|\alpha\beta\rangle + \sqrt{1-p}\,|\alpha^{\perp}\beta^{\perp}\rangle$$

(Schmidt decomposition)

$$\lambda = \sqrt{p}\,,\ \mu = \sqrt{1-p}$$

$$U^A = |\alpha\rangle\langle \mathrm{H}| + |\alpha^{\perp}\rangle\langle \mathrm{V}|,\ U^B = |\beta\rangle\langle \mathrm{H}| + |\beta^{\perp}\rangle\langle \mathrm{V}|$$

$$|\psi\rangle = U^A \otimes U^B\left(\lambda|\mathrm{HH}\rangle + \mu|\mathrm{VV}\rangle\right)$$

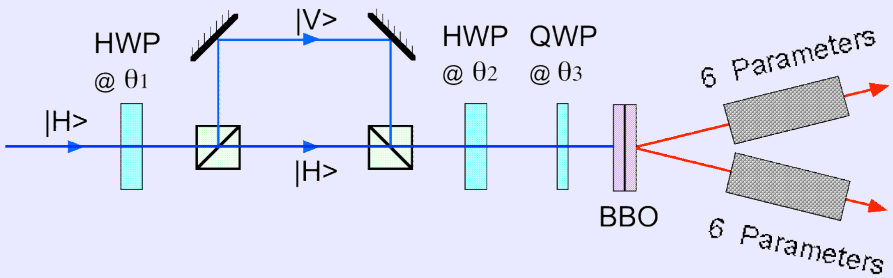
Arbitrary Two-qubit State Synthesis: Any Mixture/Purity

(work in progress)

$$\begin{pmatrix} A_1 & A_4 e^{i A_5} & \cdots & \\ A_4 e^{-i A_5} & A_2 & & \\ & & A_3 & \\ & & & 1-(A_1+A_2+A_3) \end{pmatrix}$$

15 free parameters (A_1, A_2, …)

$\Rightarrow$ 15 knobs! (θ_1, θ_2, …)



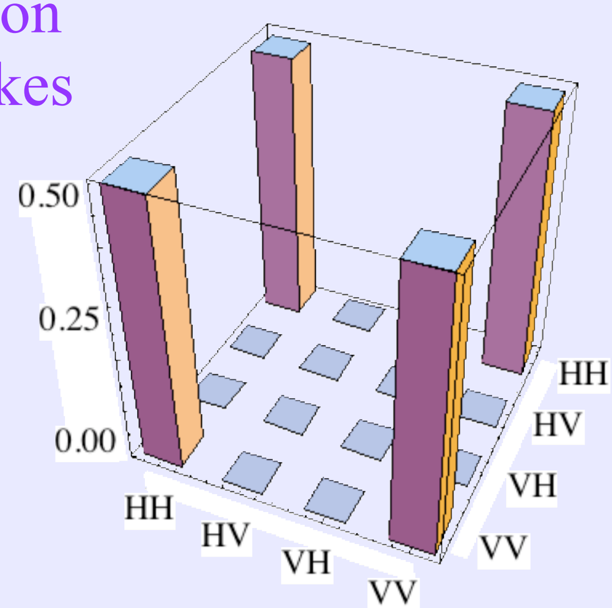
$\theta_1 = f(A_1, A_2, \ldots)$

.
.
.

Example: If we can set each of the parameters to one of 10 values, then we should be able to make 10^{15} states.

Quantum State Tomography

Measuring the density matrix for the 2-photon quantum system (c.f., measuring the Stokes parameters for a single photon)



$$\rho_{pure} = |\psi\rangle\langle\psi|$$

Example: $|\psi\rangle = \frac{(|HH\rangle + |VV\rangle)}{2}$

$$\rho = \frac{\{|HH\rangle\langle HH| + |VV\rangle\langle VV| + |HH\rangle\langle VV| + |VV\rangle\langle HH|\}}{2}$$

Experimental Tomography

Single Photon - determine the Stokes parameters
Measure contribution of H, V, D, R

Two Photon

Measure polarization correlations:

HH	HV	HD	HR
VH	VV	VD	VR
DH	DV	DD	DR
RH	RV	RD	RR

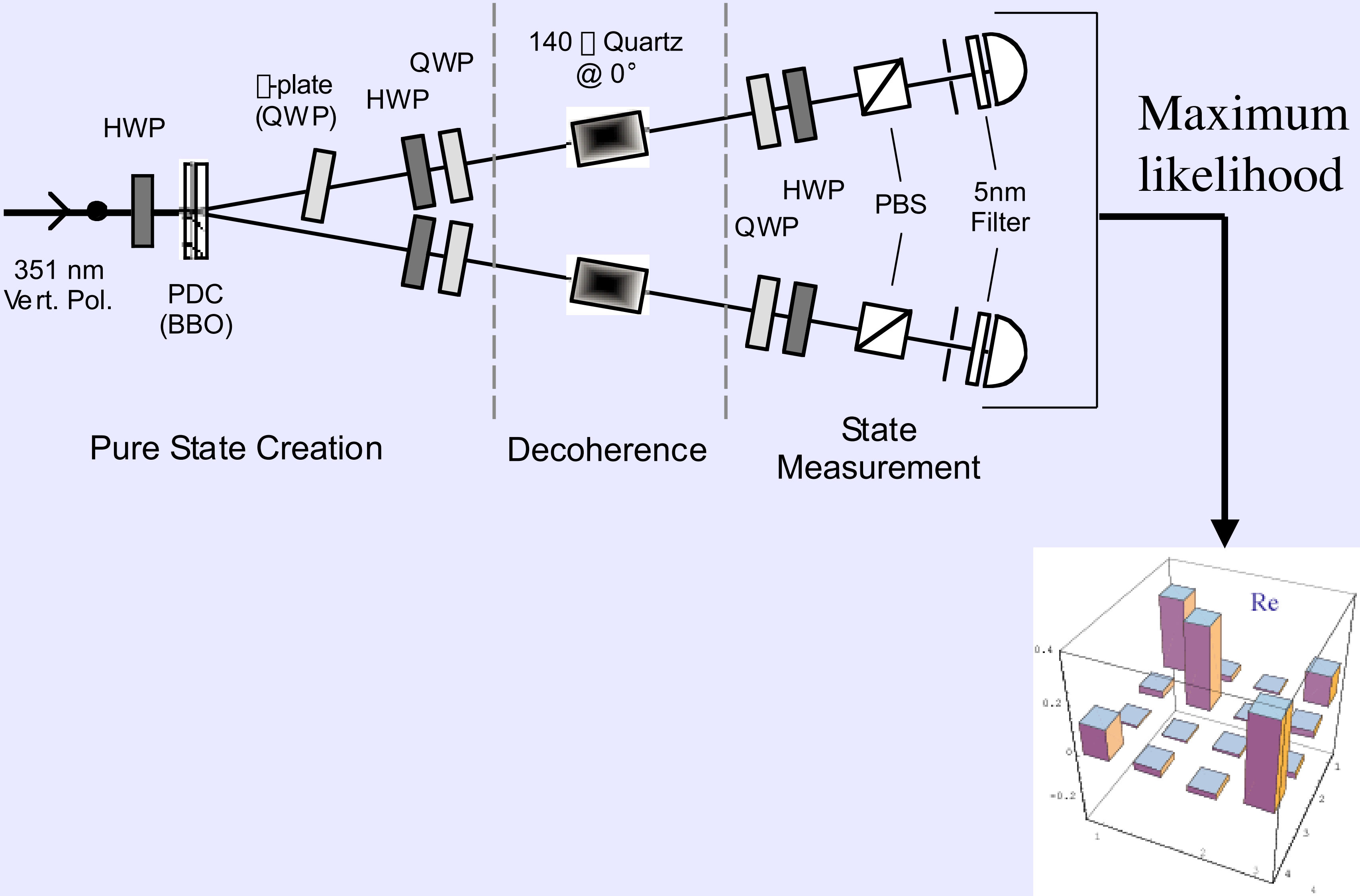
In principle, we can analytically extract the density matrix □ from these measurements.

In practice, statistical noise and setting uncertainties will typically lead to a □ with *negative* eigenvalues. ☹

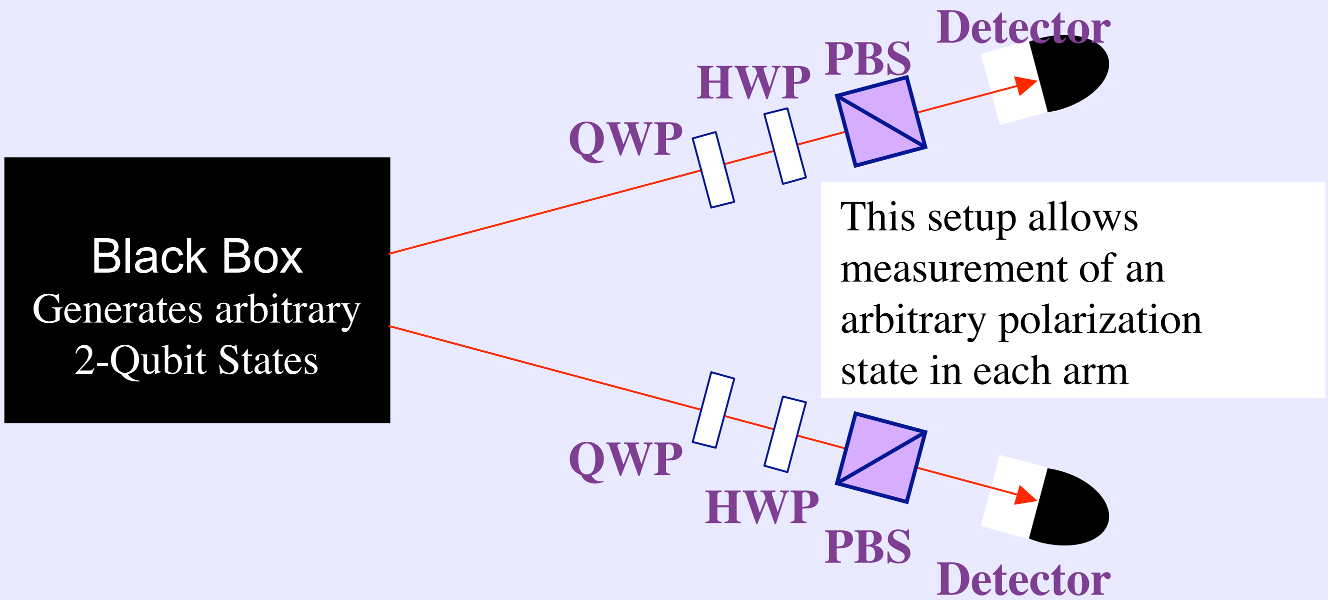
Solution: Use method of maximum likelihood to find the legitimate □ most consistent with the measured data.

[D. James et al. quant-ph/0103121]

2-qubit state creation & tomography



Computer Automated Tomography
of 2-Qubit Polarization States



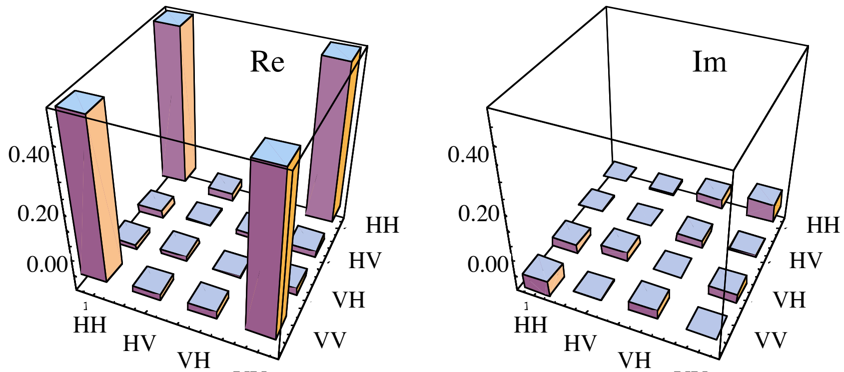
Any 2-qubit tomography requires 16 of these measurements.

Examples

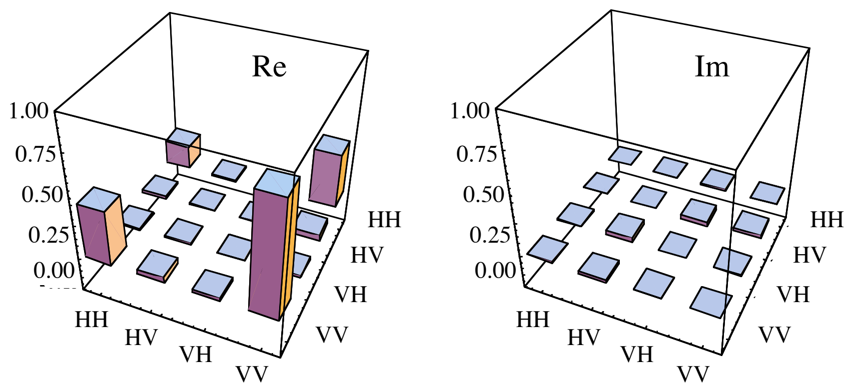
Arm 1	Arm 2
H	V
H	R
D	D

Manual (the old way)	Automated	Automated and Compensated (10 - 100x Brighter)	
		Ultra-Precise	Ultra-Fast
Manually set WP's for each measurement. (100 seconds each)	Computer sets WP's for all measurements. (100 seconds each)	Computer sets WP's for all measurements. (100 seconds each)	Computer sets WP's for all measurements. (10 seconds each)
60 minutes	30 minutes	30 minutes	**3 minutes**
Counting Error: **1.0 %**	Counting Error: **1.0 %**	Counting Error: **0.1 – 0.3 %**	Counting Error: **0.3 – 1.0 %**
WP Setting Error: **0.5 degrees**	WP Setting Error: **0.001 degrees**	WP Setting Error: **0.001 degrees**	WP Setting Error: **0.001 degrees**

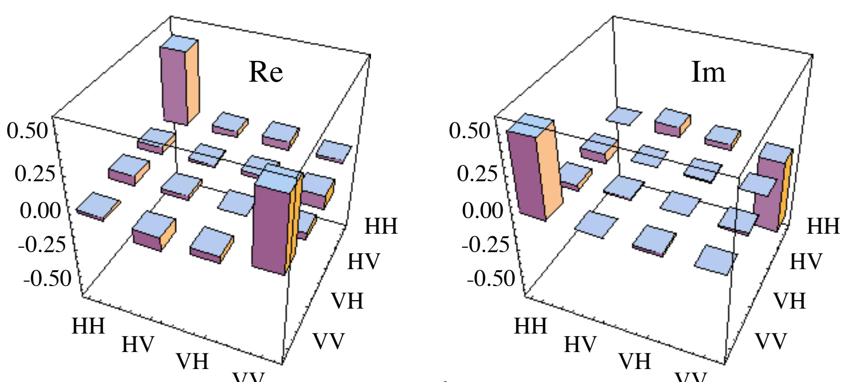
2-Photon Quantum Tomography



HH + VV

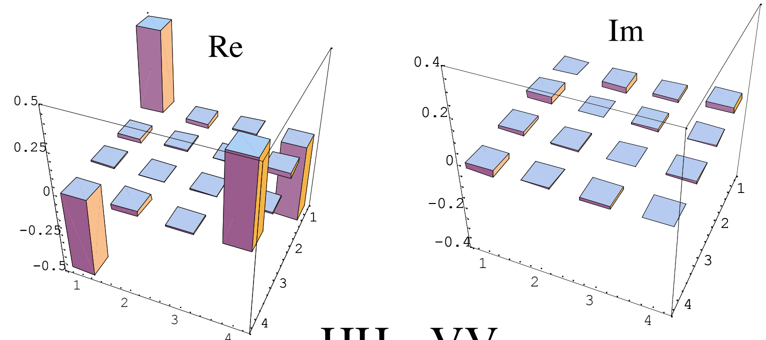


0.45 HH + VV



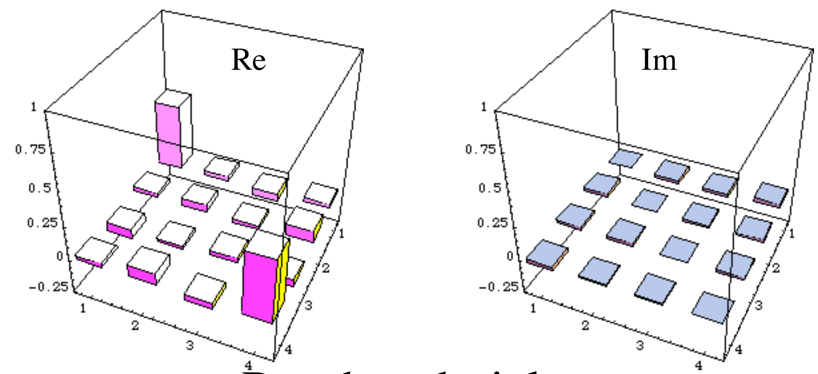
HH - i VV

From New System:



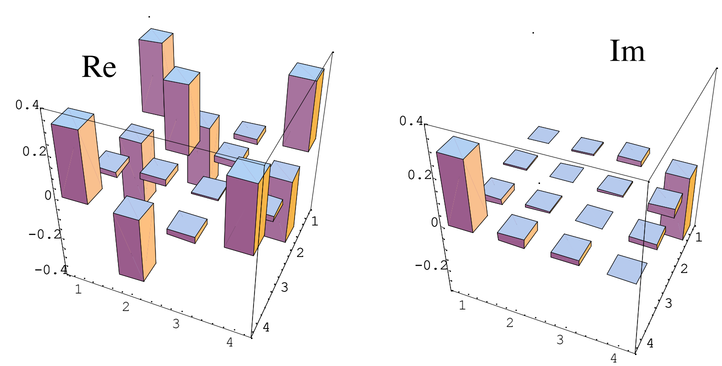
HH - VV

F = 0.983



Decohered triplet

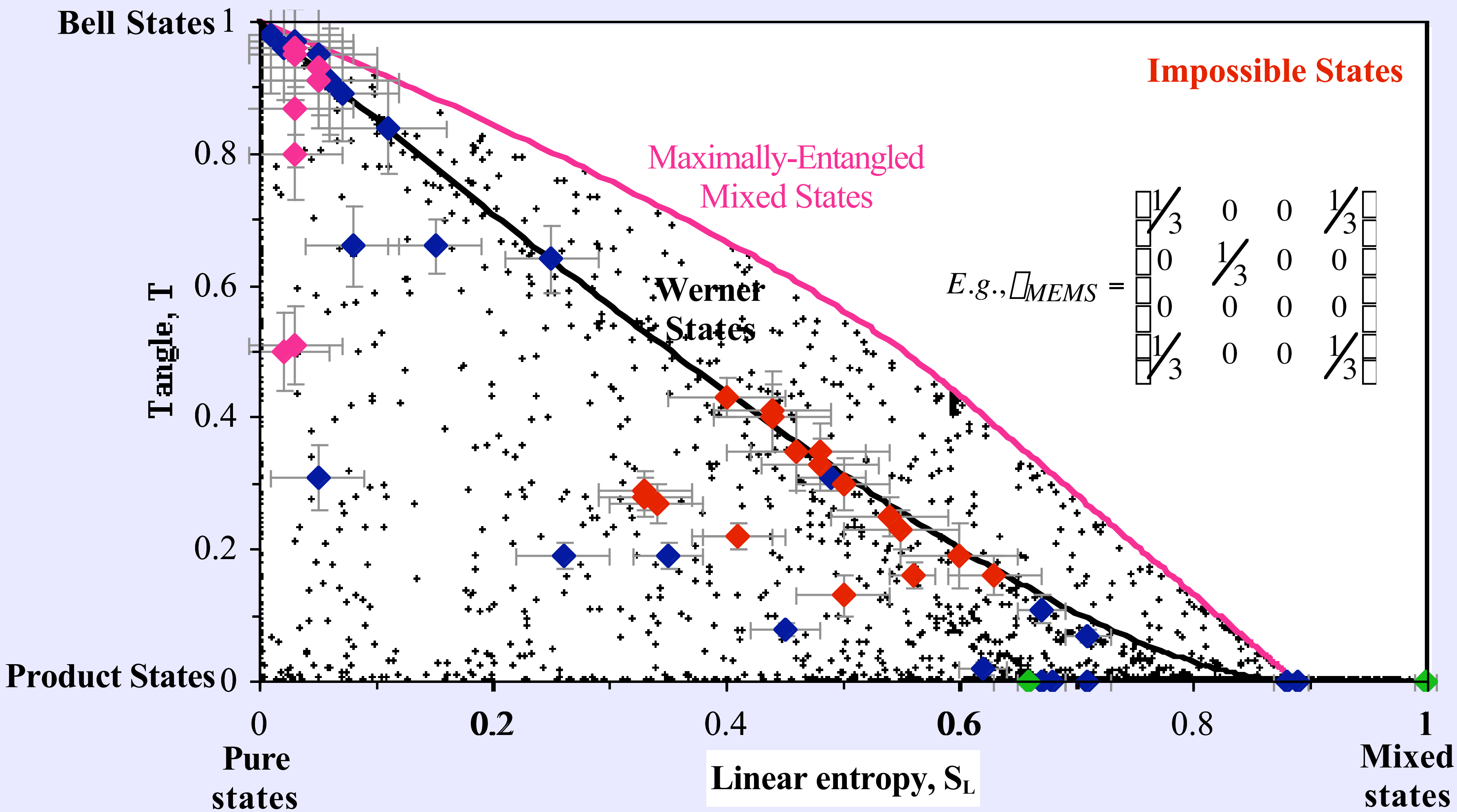
F = 0.993



Pre-MEMS state

F = 0.987

2-parameter characterization of 2-qubit states



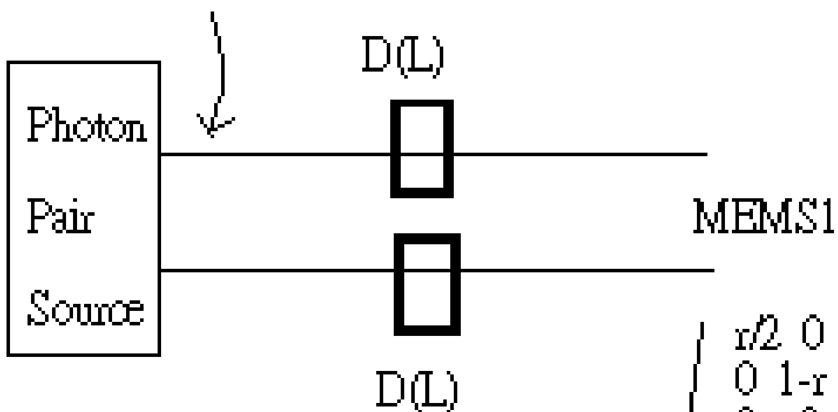
White et al., PRA **65**, 012301 (2001)

Munro et al., PRA **64**, 030302 (2001)

Making MEMS:

Creation of MEMS 1: one decoherence will do

$$|\psi\rangle = \sqrt{r/2}\,HH + \sqrt{1-r}\,HV + \sqrt{r/2}\,VV$$



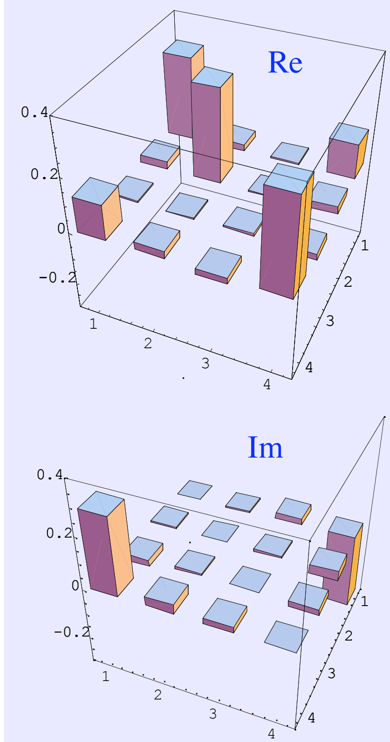
$$\begin{pmatrix} r/2 & 0 & 0 & r/2 \\ 0 & 1-r & 0 & 0 \\ 0 & 0 & 0 & 0 \\ r/2 & 0 & 0 & r/2 \end{pmatrix}$$

$\triangle$ n L << coherence length of pump

$\triangle$ n L >>coherence length of PDC pair

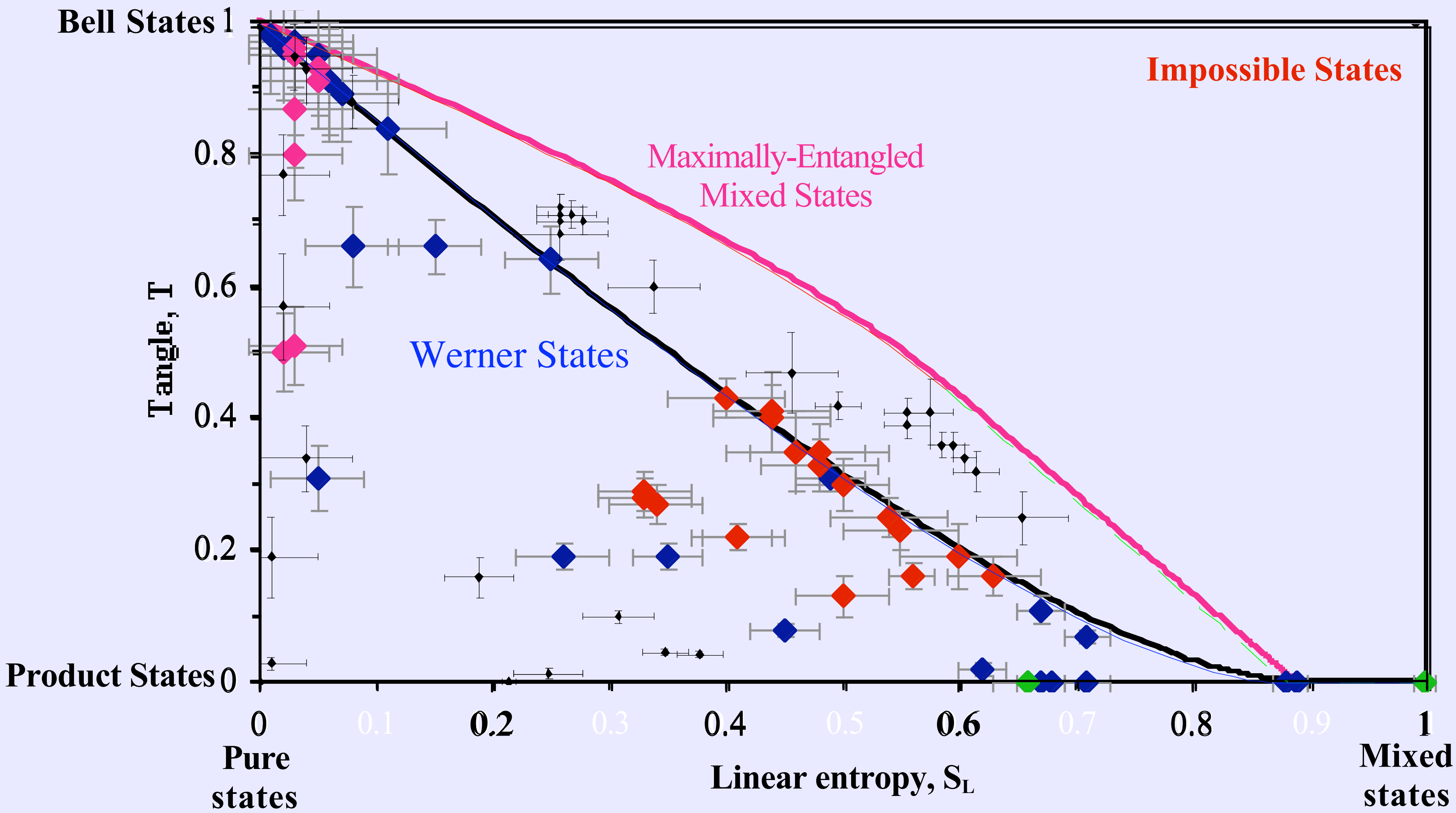
e.g., r=2/3, get you the 1/3 MEMS state

First MEMS:

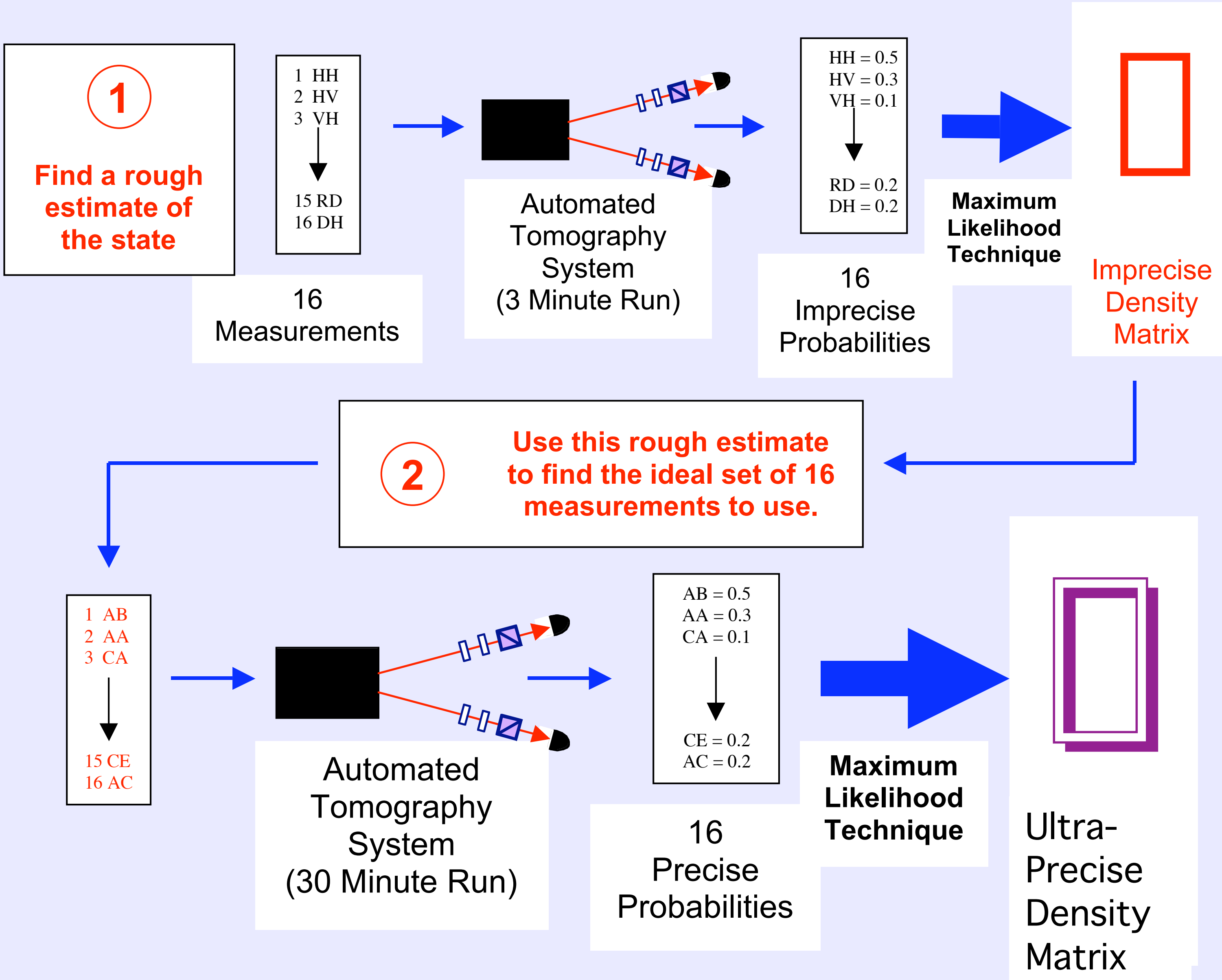


F = 0.985

Beyond Werner States…



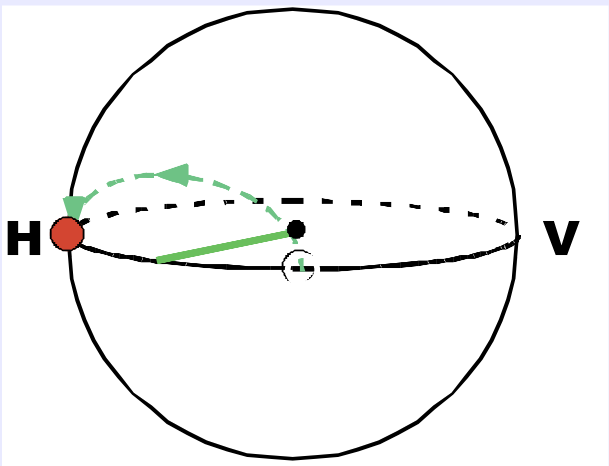
Adaptive Tomography



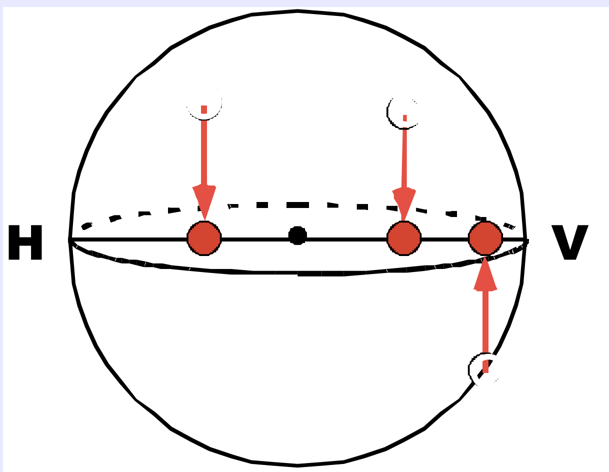
What is a quantum process?

A completely positive map, which converts an initial quantum state (possibly mixed) into a different quantum state, e.g. unitary transformations and decoherence.

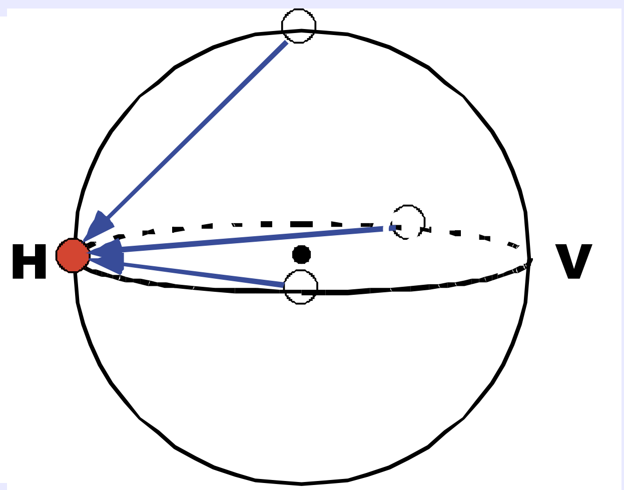
Unitary Transformation:



Decoherence:



Polarizer:

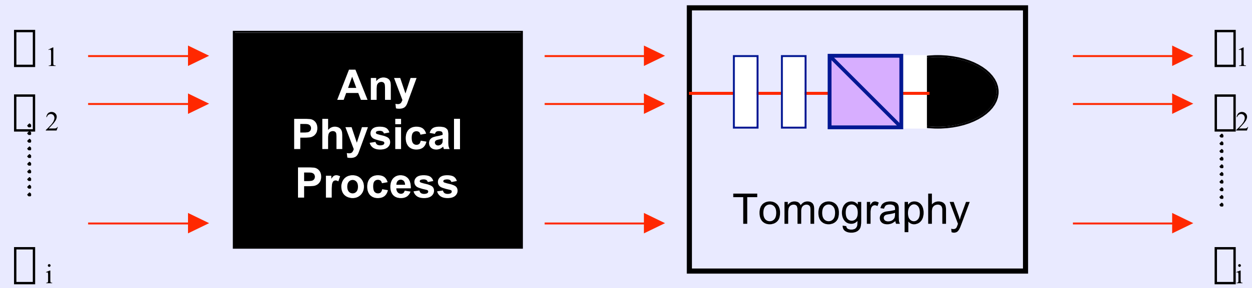


How do we characterize one?

Quantum Process Tomography

Chuang & Nielsen, JMO 44, 2455 (1997);
Poyatos, Cirac & Zoller, PRL 78, 390 (1997)

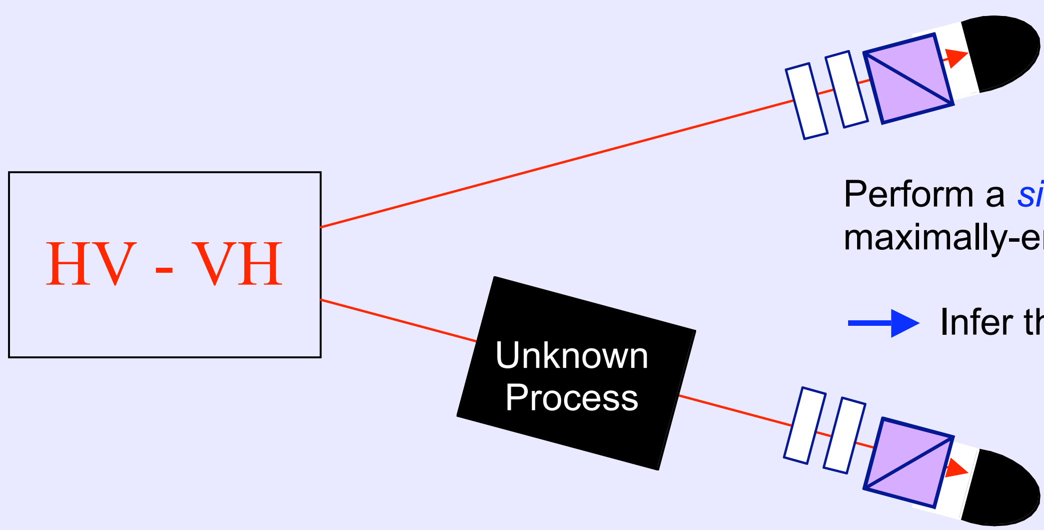
STANDARD



The □ → □ transforms allow us to reconstruct the unknown process.

ENTANGLEMENT-ASSISTED

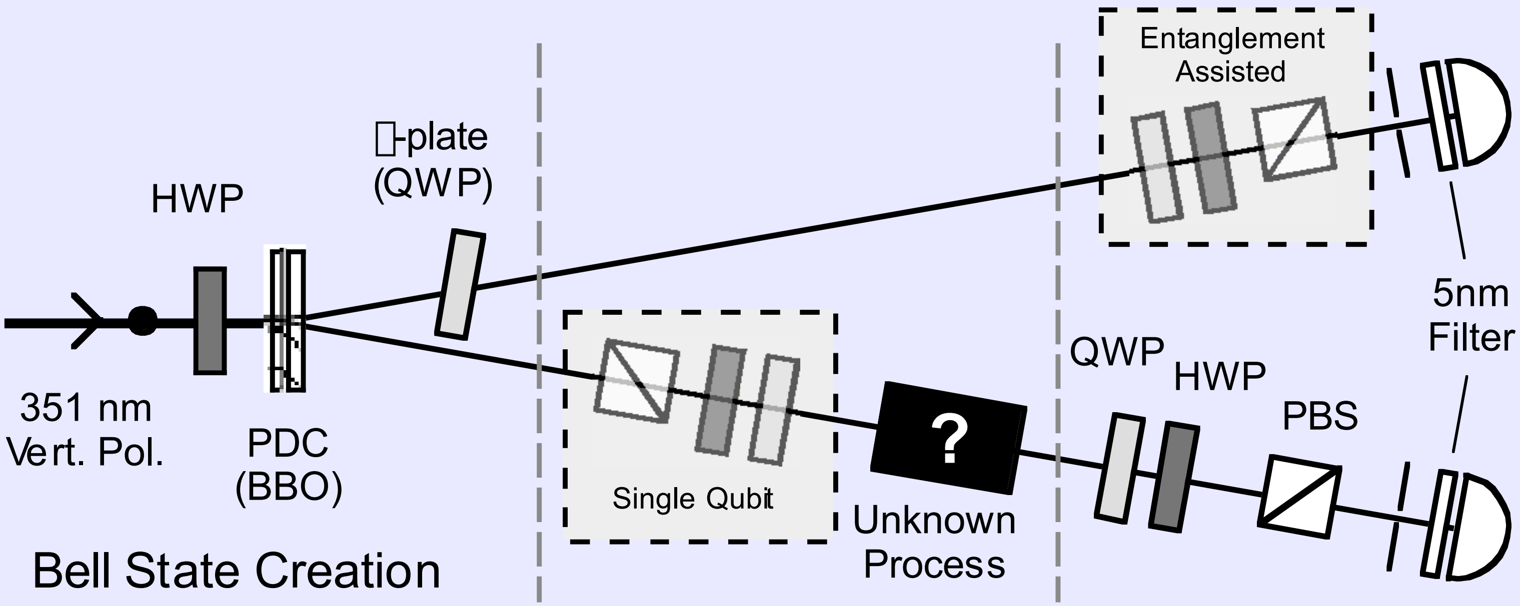
(D'Ariano & Presti; Leung; Nielsen, Thew and White)

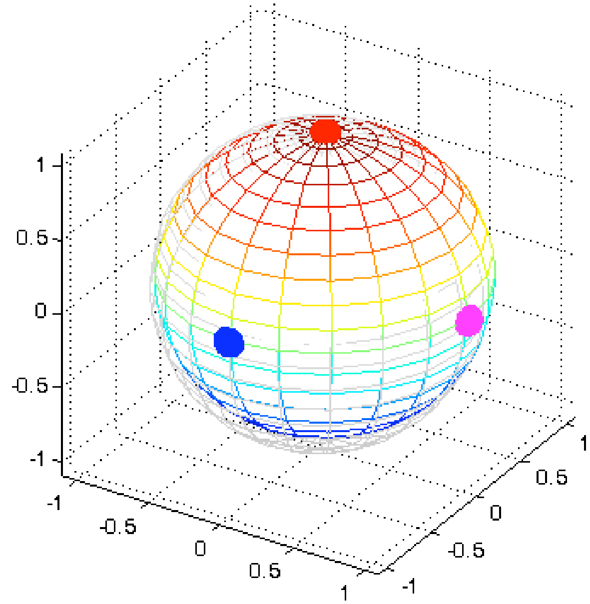
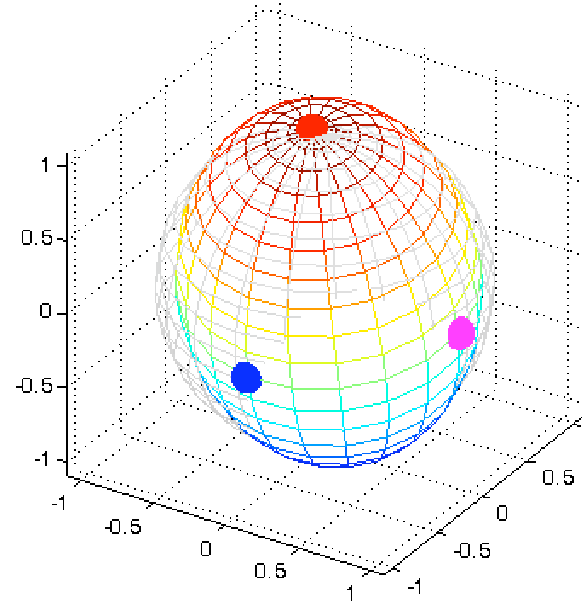
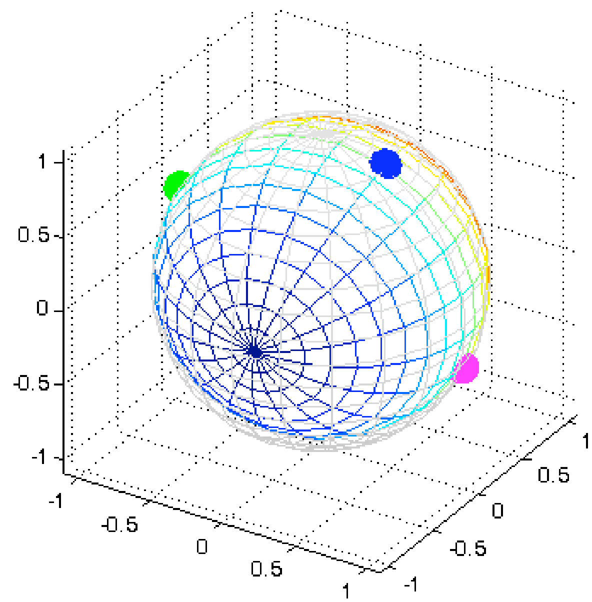
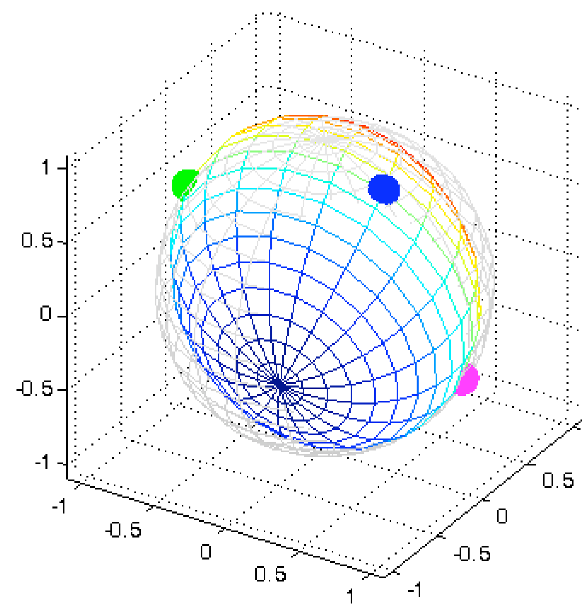


Perform a *single* tomography on a maximally-entangled input state.

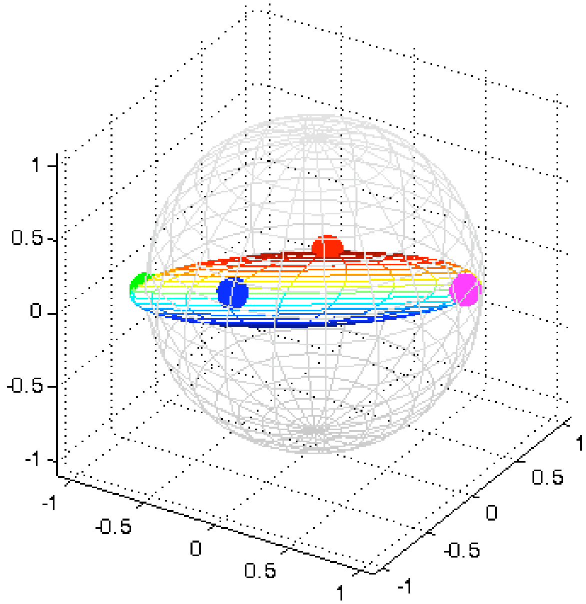
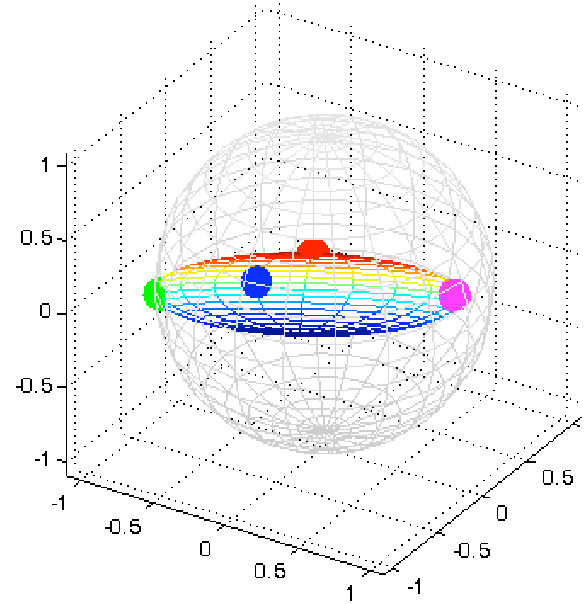
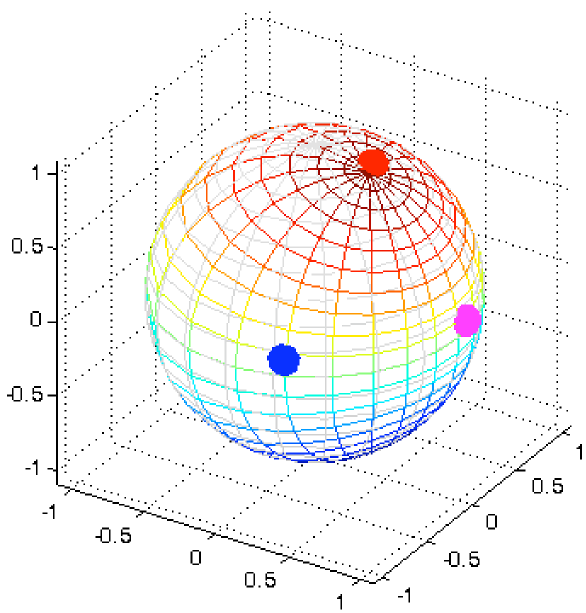
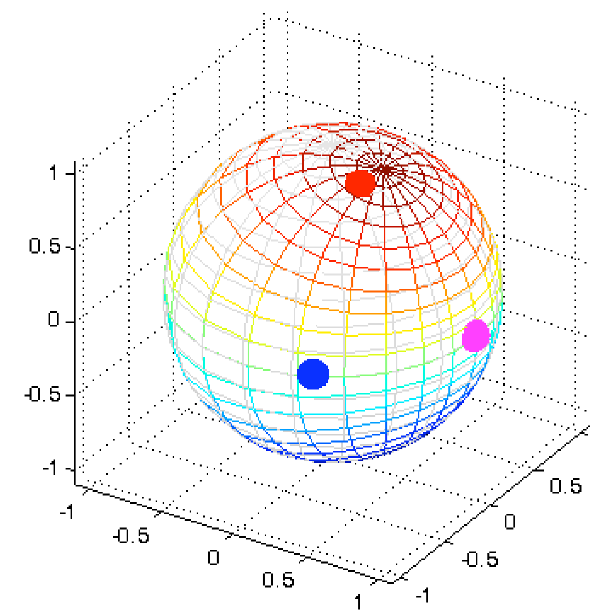
→ Infer the unknown process.

Single-Qubit & Entanglement Assisted Process Tomography



Process	Single-qubit	<F>	Entanglement-assisted
Identity:		99.2	
Rotate:		99.1	

Single-qubit | Entanglement-assisted

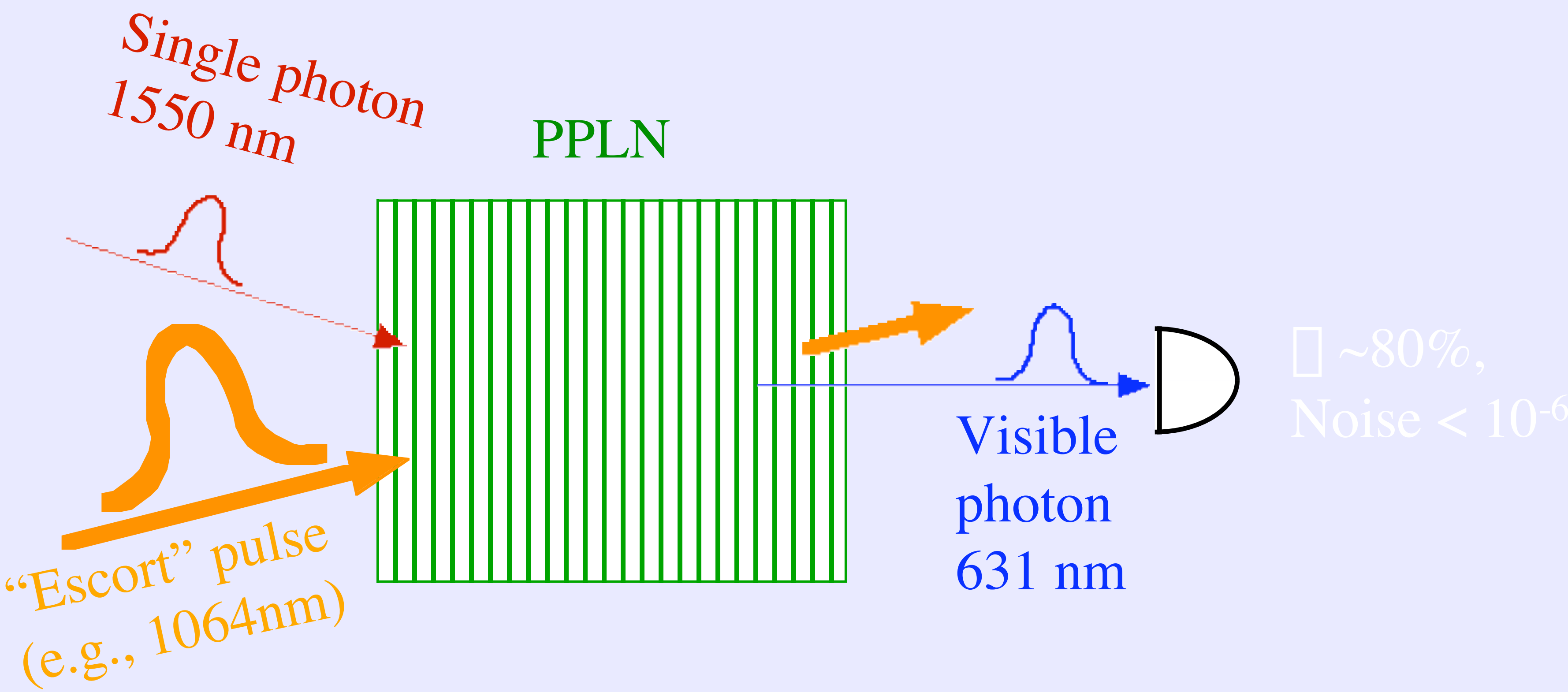
Process	Single-qubit	<F>	Entanglement-assisted
Decoherer:		99.3	
Partial polarizer:		98.7	

Quantum Transducer

Convert photon quantum state from one wavelength to another

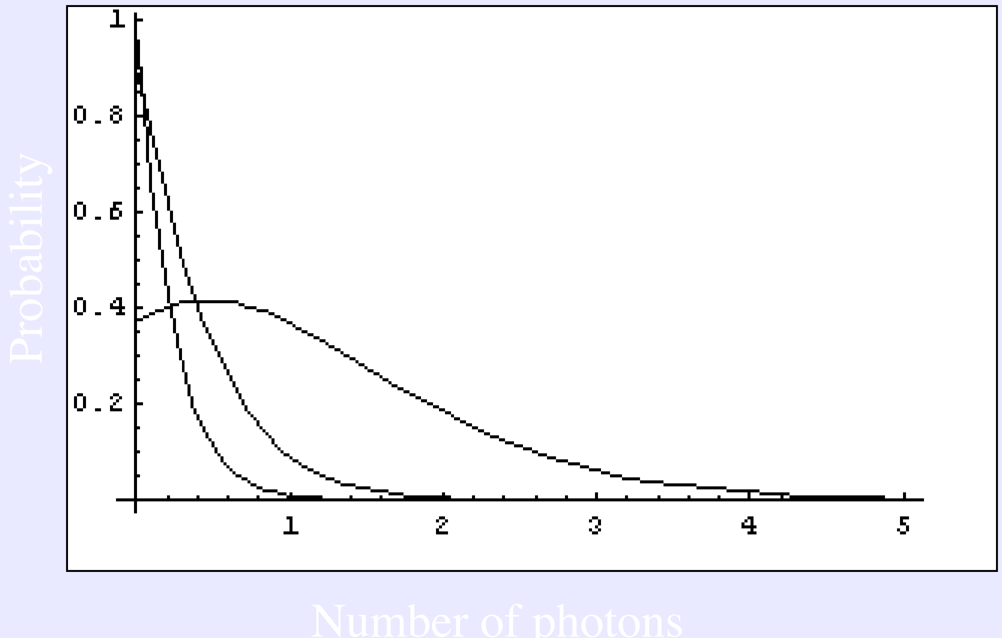
- Capitalize on high-efficiency low-noise single-photon counters (Si APDs) from 500-800nm
- Convert from (telecom) flying qubit to (atomic) storage qubit

Example:



Deterministic Single Photon Source

Classical (laser) source → Poisson photon number distribution

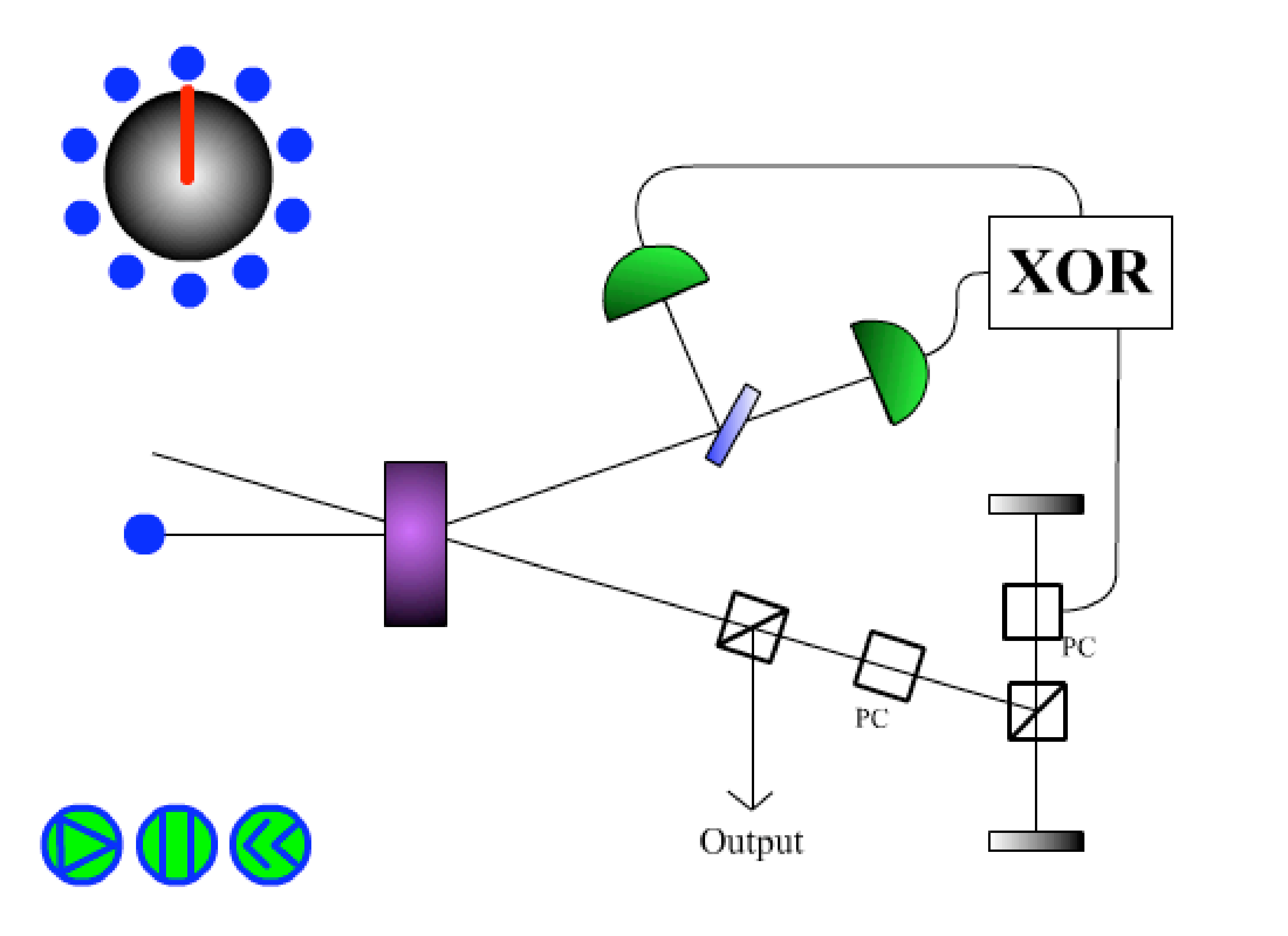


We wish to overcome this, yet retain controllable spectral characteristics

- Optical quantum computing and communication
- Quantum metrology
- Quantum cryptography

Experimental Design

We collect down-conversion from several pulses, and search for one that has exactly one pair, then store it in a delay cavity until the target time



→ single-photon output at regular intervals
(~*deterministic* output from *nondeterministic* input!)

Target efficiencies

Compare to a classical pulsed laser

37% maximum probability of making one photon
18% probability of making two photons

Current technology

70% probability of making one photon
3% probability of making two photons
10 kHz rate

Stage 2 technology (multiple attempts method)

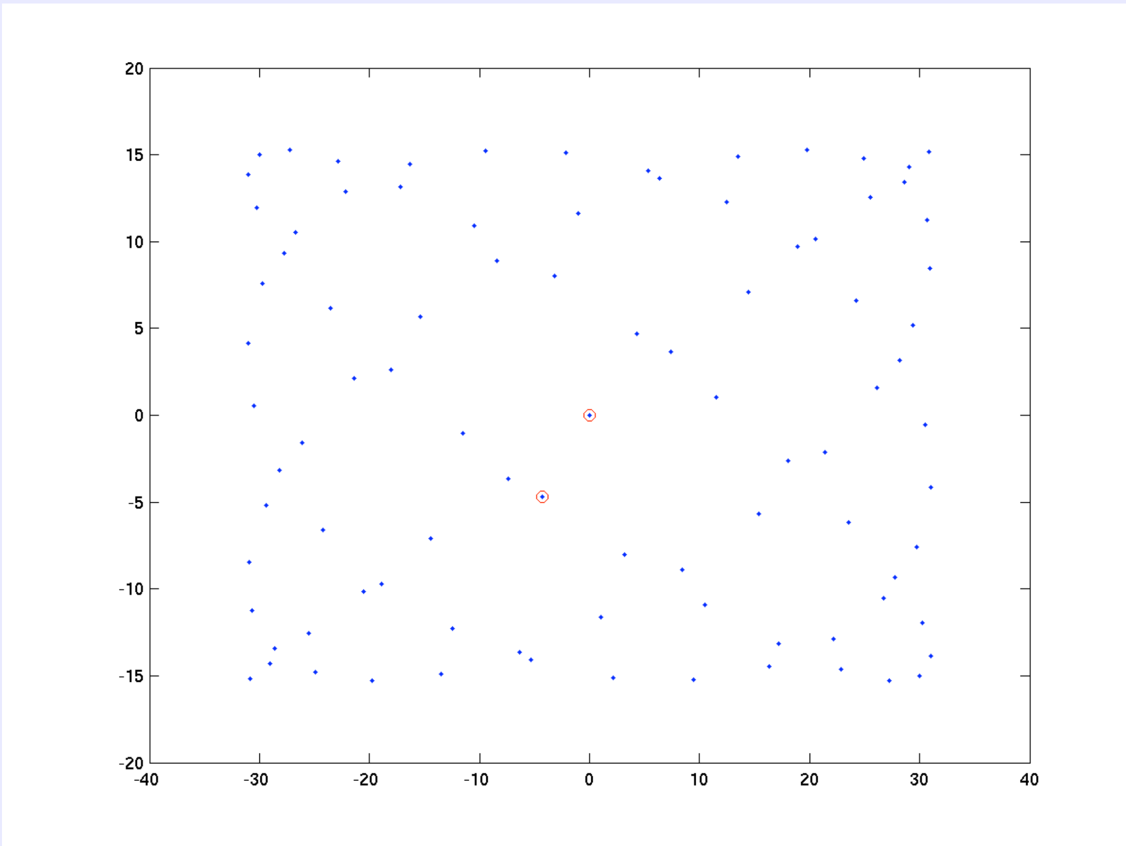
90% probability of making one photon
1.8% probability of making two photons
50 kHz rate

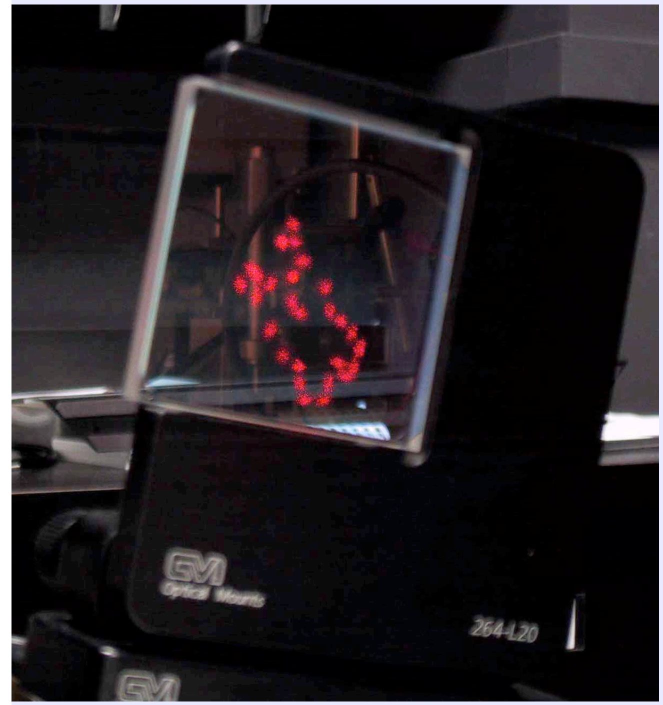
Future technology (more efficient detectors, lasers, electrooptics)

95% probability of making one photon
<1% probability of making two photons
10 MHz rate

Quantum Memory

In slow systems, the delay line may need to store photons for 1-50 μs. Using a specially designed cavity, we can achieve this.





50 pass (300 ns) cavity

This can also be used as a quantum storage in other quantum information applications.

Summary

- High-fidelity synthesis of arbitrary single-qubit states
- Bright source of tunable 2-qubit states

  Even brighter source to come

- Automated quantum tomography

  Adaptive tomography still to come

- Filling the tangle-entropy plane -- MEMS

  Arbitrary states still to come

- Single qubit/entanglement-assisted process tomography

  Arbitrary process synthesis still to come

- Also in progress:

  Quantum transducer, Single photons, Quantum memory

Kwiat's *Quantum* Clan